Exercise 24

Find the area of a triangle bounded by the y axis, the line $f(x) = 10 - 2x$, and the line perpendicular to f that passes through the origin.

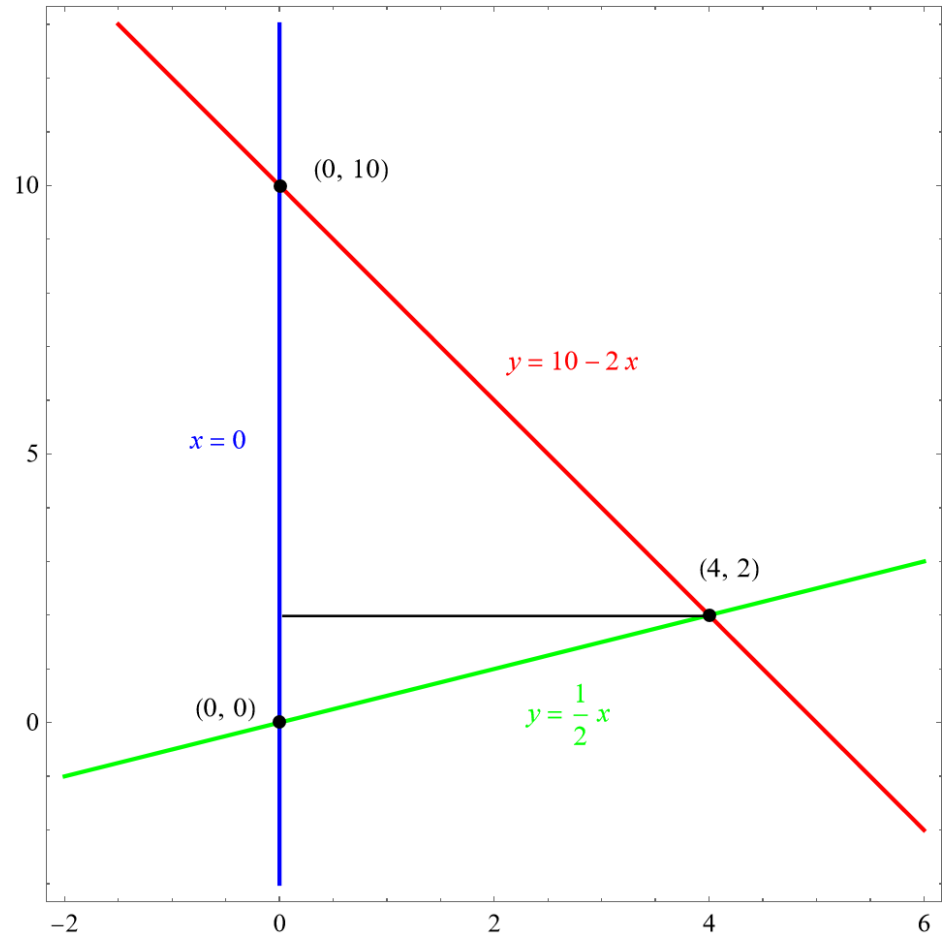
Solution

The equation for the y-axis is $x = 0$, the line $y = 10 - 2x$ is given. The line perpendicular to f has a slope of $1/2$. Use the point-slope formula to get the equation of this line.

$$y - 0 = \frac{1}{2}(x - 0)$$

$$y = \frac{1}{2}x$$

Graph all three lines.



The area of the triangle is

$$A = \frac{1}{2}bh = \frac{1}{2}(4)(10) = 20.$$

The point of intersection on the right is found by setting the two equations for y equal to each other.

$$10 - 2x = \frac{1}{2}x$$

$$10 = \frac{5}{2}x$$

$$\frac{20}{5} = x$$

$$x = 4$$

Plug this value of x into either of the two equations for y to get

$$y = 10 - 2(4) = 2.$$

This means the point of intersection on the right is $(4, 2)$.